The non-equivalence of Weyl and Majorana neutrinos with standard-model gauge interactions

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Abstract

It is shown that a Majorana neutrino with identical phenomenology as a standard-model Weyl neutrino obeys a Lagrangian different by a factor $\sqrt{2}$ in the weak-interaction term from the one that follows from the standard model. Assuming that the standard model *does* hold in good approximation for the weak interactions of a Majorana neutrino e.g. a charged-current production cross section a factor 2 larger than the observed one is predicted. From this it is concluded that the neutrino is not of Majorana type. This conclusion does *not* forbid the possible existence of a Weyl neutrino with a Majorana mass term and ensuing lepton-number violating phenomena (like e.g. neutrino-less double beta decay). The paper is not in contradiction with any published literature, because it analyses the formal proof of equivalence between Weyl and Majorana neutrino for the first time under the assumption that the standard model is *quantitatively* correct for the weak interactions of the neutrino.

1 Introduction

1.1 A remark about the meaning of the present paper's conclusion

In section 4 of this paper I show that the observed neutrinos - in the massless limit - are definitely of Weyl and not Majorana type. This conclusion only holds under the following two, widely used, *assumptions*:
A.the standard model *quantitatively* describes the weak interactions of the neutrino to good approximation
B. standard quantum field theory applies to good approximation for the description of the neutrino field

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As discussed in section 5 my conclusion holds also if Weyl neutrinos have finite Majorana masses. This is why the present manuscript's claim is not as far reaching as it might first sound: all of the lepton-number violating phenomena connected with massive Majorana neutrinos, such as neutrino-less double beta decay and all Majorana mass related small-mass generating mechanisms proposed for them (such as the see-saw mechanism) remain viable for a *Weyl neutrino with a Majorana mass term*. On the other hand - because the term "Majorana neutrino" is precisely mathematically defined (section 2) - the manuscript's conclusion is nontrivial (it solves a longstanding question of neutrino physics) and potentially important in the development of new theories.

1.2 Outline of the paper

The review of the precise definition of the Majorana-neutrino field in section 2 makes clear that *mathematically* Weyl and Majorana fields are different. Langrangians predicted by the standard model for two mathematically different fermion fields - even if they have the same degrees of freedom - must not necessarily lead to the same phenomenology[1]. Below I derive the Lagrangians giving rise to the same phenomenology (briefly outlined in the next paragraph "1.") and the Lagrangians derived from the standard model (briefly outlined in following paragraph "2.") and find they are *not* identical.

1. In section 3 - as a review of work from several authors in the late 1950s - it is shown that a unitary similarity transformation (the "Pauli I" transformation) between Weyl and Majorana neutrino fields exists. One concludes that Weyl neutrinos with a standard-model (SM) Lagrangian $\mathrm{L}_{\mathrm{Weyl}}^{\mathrm{SM}}$ can be unitarily transformed to a Majorana neutrino with a Lagrangian $\mathrm{L}_{\mathrm{Maj}}^{\mathrm{Weyl-equivalent}}$ which is uniquely specified by the condition that any similarity transformation leaves the form of all field equations unmodified. A Majorana neutrino *obeying this Lagrangian* is then phenomenologically completely equivalent to a Weyl neutrino.
2. Since the late 1970s the interaction of the left-handed Weyl neutrino ν_L is uniquely specified by the standard model *without any reference to the observed neutrino's properties*, (analogous to the top quark, whose weak-interaction properties were all specified before its actual discovery). Because a Majorana-neutrino field can be decomposed to Weyl-neutrino components via the "Pauli I" transformation, one can derive the standard-model Lagrangian of a Majorana neutrino $\mathrm{L}_{\mathrm{Maj}}^{SM}$. One finds that $\mathrm{L}_{\mathrm{Maj}}^{\mathrm{SM}} \neq \mathrm{L}_{\mathrm{Maj}}^{\mathrm{Weyl-equivalent}}$ In other words: the "Pauli I" transformation - that brings a Weyl to a phenomenologically equivalent Majorana neutrino - is not SU(2) invariant.

If the observed neutrino were of Majorana type it *had* to be phenomenologically equivalent to a Weyl neutrino with standard-model interactions to not contradict experimental facts (e.g. about neutrino cross sections). Under assumption A. in the introduction we can then conclude that the observed neutrino must be of Weyl type. The end of section 3 discusses why this conclusion is not in contradiction with the pubished literature on the "Majorana

[1] An example where this is obvious is e.g. a Weyl neutrino and a massless electron.

Dirac confusion theorem".
Section 5 of the paper is devoted to showing in detail how a Weyl neutrino with a Majorana mass term can violate lepton number without ever being its own anti particle and section 6 concludes.

2 The definition of a Majorana-neutrino field

For definitness I first discuss the quantitative definition of a "Majorana neutrino field". In this I follow the standard literature. Let ν be a 4-component Dirac neurino field[2] Then, using "east-coast" notation (imaginary time) the 2-component Weyl field ν_L is defined[3] as[1]:

$$(1) \quad \nu_L = \frac{(1+\gamma_5)}{2}\nu_L = P_L \nu_L$$

The 2-component Majorana field ν_M is defined via the following relations by which the neutrino is its own antiparticle [2]:

$$(2) \quad \nu_M = C{\nu_M}^{\dagger T}\gamma_4 \equiv \nu_M^c;\ \nu_M = \frac{1}{\sqrt{2}}(\nu + \nu^c)$$

Here C is the charge conjugation matrix, $\dagger$ the hermitian conjugate and T a transpose acting only on the spinor, ^{c} symbolizes charge conjugation and a conventional "creation phase factor"[3] was set to 1 [4]. The definition in eq.(2) after the semicolon defines the field normalization and it can be easily shown to be the one that fulfills the usual field-anticommutation axioms of quantum-field theory:

$$(3) \quad \left[\nu_M(\vec{x}, t), \eta(\vec{x}', t)\right]_+ = i\delta(\vec{x} - \vec{x}')$$

where η is the field which is canonical conjugate to ν_M. This field normalisation (*with* the factor $1/\sqrt{2}$) is generally used in the literature[3, 8, 9, 10, 11] [5].
Clearly the conditions eq.(1) and eq.(2) are mathematically mutually exclusive; a Weyl particle can never be its own antiparticle.

[2] I do not discuss the possible case of Dirac neutrino masses in this paper.

[3] I do not explicitely include the condition of fermion number conservation in this definition, as is sometimes done in the literature.

[4] This defining constraint for Majorana fields ("Majorana neutrinos are eigenstates of C") is universally accepted in the literature, also after the discovery of parity violation (e.g. eq.(19) in Pauli's paper[4] written after the discovery of parity violation, or e.g. eq.(10) in Ref.[5]). Even if one prefers Kayser's alternative characterization of Majorana neutrino as a state that is turned into itself with reversed helicity under CPT [3], eq.(2) has to hold. E.g. in a textbook by Mohapatra and Pal [6] the CPT properties of Majorana neutrinos are derived in section 4.4.3 *using* their eq.(4.16) which is identical to eq.(2). Berestetskiĭ et al.[7] argue that there is no problem with condition (2) even in the presence of weak interactions because it is invariant not only to CPT but also with respect to each of these transformations seperately.

[5] Only Bilenky and Petcov[12] leave the factor $1/\sqrt{2}$ out, for reasons that are not clear (they quote Ref.[8] that *does* use it as standard reference for the definition of a Majorana fermion.)

3 The unitary equivalence of Weyl and Majorana neutrino fields, also called "Dirac-Majorana confusion theorem"

For the present argument it is sufficient to consider only charged currents in the low energy limit. The standard-model (SM) Lagrangian for massless Weyl neutrino fields is then:

$$(4) \quad L^{\rm SM}_{\rm Weyl} = \bar{\nu}_L \gamma_\mu \frac{\partial}{\partial x_\mu} \nu_L + ig/\sqrt{2}\left[W^-_\mu \bar{e}_L \gamma_\mu P_L \nu_L + H.C.\right]$$

here $g = \frac{e}{sin(\theta_w)}$. Let us now answer the following question: What Lagrangian "$L^{\rm Weyl-equivalent}_{\rm Maj}$" must hold Majorana neutrino, so that it shows a phenomenology identical to the one of the Weyl neutrino with Lagrangian (4)?

Pauli[4] specified the following "Pauli I" transformation "U_1" which transforms a neutrino field ν into ν':

$$(5) \quad \nu' = U_1 \nu U_1^{-1} = \frac{1}{\sqrt{2}}(\nu - \gamma_5 \nu^c)$$

This transformation [6] can be easily shown to be *unitary* but does not conserve a SU(2) invariance of a Lagrangian. Similarity transformations leave the form of operator equations (i.e. in particular the field equations and anticommutation relations) unmodified and the expectation values of field operators do not change under a unitary transformation of field operator together with the field states[11, 4]. Therefore the phenomenology remains unchanged if one replaces ν by ν' everywhere.

For the special case $\nu = \nu_L$ eq.(5) reads (h=helicity)[7]:

$$(6) \quad \nu' = \frac{1}{\sqrt{2}}(\nu_L - \gamma_5 (\nu_L)^c) = \frac{1}{\sqrt{2}}(\nu_L + (\nu_L)^c) = \nu_M(h = -1)$$

From the invariance of the field equations, the Majorana Lagrangian is obtained by replacing ν_L with ν_M in equation (4)

$$\begin{aligned} L^{\rm Weyl-equivalent}_{\rm Maj} &= \bar{\nu}_M(h = +1)\gamma_\mu \frac{\partial}{\partial x_\mu} \nu_M(h = -1) \\ (7) \qquad & +ig/\sqrt{2}\left[W^-_\mu \bar{e}_L \gamma_\mu P_L \nu_M(h = -1) + H.C.\right] \end{aligned}$$

The "Pauli I" transformation eq.(5) does not include the electron field and is therefore not equivalent to a mere representation change of the theory. It is thus not at all clear if this Lagrangian still obeys the standard model (see next section). In the late 1950s (i.e. long before the formulation of the standard model) - with no reason whatsoever to exclude the validity of $\mathrm{L}^{\rm Weyl-equivalent}_{\rm Maj}$ for neutrinos - various authors [13, 14, 15, 16, 17, 18, 19] could only conclude that massless ν_L and ν_M (helicity=-1 states) (and analogously $\bar{\nu}_L$ and $\bar{\nu}_M$ (helicity=$+1$ states)) are phenomenologically completely equivalent (this conclusion was

[6] The explicit form of U_1 can be found in Refs.[13, 9]

[7] The Pauli I transformation of ν_L is $U_1 \nu_L U_1^{-1} = U_1 P_L \nu U_1^{-1} \neq P_L(U_1 \nu U_1^{-1})$

later also called "Dirac - Majorana confusion theorem"[3]). The "Dirac - Majorana confusion theorem" was never discussed in the literature under the assumption of *quantitative* validity of the standard model. The difference between Majorana and Weyl neutrino is of a *purely* quantitative character (a factor $\sqrt{2}$) all qualitative properties are the same (e.g. in the massless case both Majorana and Weyl neutrinos conserve lepton number). Kayser[3] and Zrałek[20] state the confusion theorem's validity under the assumption that the weak interaction is left handed ("qualitative validity" of the standard model), a correct statement which is not in contradiction with the present paper.

4 Proof that $\mathbf{L}_{\text{Maj}}^{\text{Weyl-equivalent}} \neq \mathbf{L}_{\text{Maj}}^{\text{SM}}$

Let us calculate the Lagrangian $\mathrm{L}_{\text{Maj}}^{SM}$ for massless Majorana neutrinos that is predicted by the Standard model. Applying P_L onto eq.(6) one gets:

$$(8) \quad \nu_L = \sqrt{2} P_L \nu_M(h=-1)$$

One can also show that [21, 22]:

$$(9) \quad \bar{\nu}_L \gamma_\mu \frac{\partial}{\partial x_\mu} \nu_L = \bar{\nu}_M(h=+1) \gamma_\mu \frac{\partial}{\partial x_\mu} \nu_M(h=-1)$$

Replacing the kinetic term in Lagrangian (4) using eq.(9) and ν_L in the interaction term using eq.(8) one gets:

$$(10) \quad L_{\text{Maj}}^{\text{SM}} = \bar{\nu}_M(h=+1) \gamma_\mu \frac{\partial}{\partial x_\mu} \nu_M(h=-1) + ig \left[W_\mu^- \bar{e}_L \gamma_\mu P_L \nu_M(h=-1) + H.C. \right]$$

The charged-current coupling constant in eq.(10) is seen to be a factor $\sqrt{2}$ larger than in eq.(7) the two Lagrangians are thus different.
The numerical value $g = \frac{e}{sin(\theta_w)}$ is determined in the standard-model gauge theory by considering only neutral-current (for $sin(\theta_w)$) and electromagnetic (for e) reactions of the electron, i.e. without reference to neutrino properties. One numerically different coupling constant in the two otherwise identical Lagrangians eq.(7) and eq.(10) is a difference which persists to the phenomenological level (i.e. the application of Feynman rules). In other words: if the neutrino is a Majorana particle and its gauge interactions are the one of the standard model, charged-current reactions of the neutrino would have a factor 2 larger cross section than observed. If we assume the strict validity of the standard model *gauge sector* a priori (see assumption A in the introduction) the observed neutrino, if massless must be a Weyl neutrino, i.e. definitely not its own antiparticle. This conclusion rests **only** on the quantitative consideration of the charged current "source" term $igW_\mu^- \bar{e}_L \gamma_\mu \nu_L$; as long as only kinetic, mass and the form of the interaction term are considered (*as is done in all equivalence proofs in the literature!*) Majorana and Weyl fields are seen to be completely equivalent.

5 Weyl neutrinos with Majorana masses

It could be that Lagrangian (4) for a Weyl neutrino contains a small Majorana mass term. In this section I first review the formal reason why this leads to a lepton-number violating theory and then analyze how this is possible for a particle that is never its own antiparticle. Lepton conservation is induced, according Noether's theorem, by the invariance of (4) under the following continuous transformation group. The charged lepton field e and neutrino field ν are simultaneously transformed via[11]:

$$\nu' = e^{i\alpha}\nu$$
$$\bar{\nu}' = \bar{\nu}e^{-i\alpha}$$
$$e' = e^{i\alpha}e \tag{11}$$

Considering α infinitesimal for the infinitesimal field transformation $\delta\Psi$, Noether's theorem yields lepton conservation. The standard model Lagrangian with the addition of a non standard-model Majorana mass term m_{Maj}:

$$L_{\rm Weyl}^{\rm SM} = \bar{\nu}_L\gamma_\mu\frac{\partial}{\partial x_\mu}\nu_L + ig/\sqrt{2}\left[W_\mu^-\bar{e}_L\gamma_\mu\nu_L + H.C.\right] + [m_{Maj}\bar{\nu}_L(\nu_L)^c + H.C.] \tag{12}$$

The treatment of section 3 continues to hold. This means:
1. Lagrangian eq.(12) is phenomenologically equivalent to the Lagrangian

$$\begin{aligned} L_{\rm Maj}^{\rm Weyl-equivalent} &= \bar{\nu}_M\gamma_\mu\frac{\partial}{\partial x_\mu}\nu_M + ig/\sqrt{2}\left[W_\mu^-\bar{e}_L\gamma_\mu\nu_M(h=-1) + H.C.\right] \\ &\quad + [m_{Maj}\nu_M\bar{\nu}_M + H.C.] \end{aligned} \tag{13}$$

2. assuming the validity of the standard-model gauge sector the neutrino is definitely not a Majorana field.
m$_{Maj}$ violates the invariance of eq.(12) under the transformation group eq.(11) because the Majorana mass term acquires a phase of e$^{2i\alpha}$ under transformation (11).
What is the mechnanism with which a Weyl field, which is never its own charge conjugate, violates lepton number? Consider the state of a Weyl field with a Majorana mass in an inertial frame at which it is a rest:

$$\gamma_4 i\frac{\partial}{\partial t}\nu_L(\text{rest}) = \mathrm{m_{Maj}}(\nu_\mathrm{L})^\mathrm{c}(\text{rest}) \tag{14}$$

A solution to this equation in the Weyl representation is: $\nu_L(\text{rest}) = \begin{pmatrix} 0 \\ \phi_0 + i\sigma_2\phi_0^* \end{pmatrix}$ with $\phi_0 = ae^{imt}$. This can be rewritten as:

$$\nu_L(rest) = \frac{1}{\sqrt{2}}(\nu_D(\text{rest}) + \nu_\mathrm{D}(\text{rest})^\mathrm{c})_\mathrm{L} = \nu_\mathrm{L} + (\nu^\mathrm{c})_\mathrm{L} \tag{15}$$

where $\nu_D(\text{rest}) = \begin{pmatrix} \phi_0 \\ \phi_0 \end{pmatrix}$ describes a Dirac particle at rest. This result means: in the rest frame the Weyl spinor consists of the components "left-handed neutrino ν_L" (helicity $= -1$)

and "left-handed antineutrino $(\nu^c)_L$" (helicity=+1) which are *not* their respective charge conjugates. A Lorentz boost along the z-axis can be shown to transform ν_L(rest) (with helicity=0) into a state which is predominantly ν_L (helicity=-1) or $(\nu^c)_L$ (helicity=+1). I.e. depending on the inertial frame, a massive Weyl particle is predominantly particle or antiparticle. In no frame it is its own antiparticle, however.
With a Majorana mass term a Weyl field can thus violate lepton conservation, *without being its own antiparticle.*

6 Summary

Within the present framework of field theory, a theory "A": "the neutrino is a Majorana particle and its weak-interaction is characterised by Lagrangian eq.(7) (which is different from the one expected in standard model)" and theory "B": "the neutrino is a Weyl particle and the standard-model gauge sector is strictly valid" are phenomenologically completely equivalent. Therefore - without a powerful theory like the standard model that quantitatively predicts the form of the neutrino weak interaction *without any recourse to measured neutrino properties* - to go from theory "A" to "B" is merely a change of designations. This is how the equivalence between Weyl and Majorana neutrinos became conventional wisdom. However, 25 years of impressive experimental confirmations of the standard model convinced most particle physicists that the gauge sector of future theories is quantitatively described by this theory to good approximation. Under this - now very plausible - assumption, theory "B" is realized in nature, i.e. Majorana's idea of hermitian fermion fields describing neutral fermions is not realized in nature for the neutrino. This is a nontrivial constraint on all future theories. I do not claim that a Majorana theory is inconsistent in any sense: I only say that experimental results happen to prefer a Weyl neutrino, without offering any theoretical reason why this should be so. The present paper does not contradict any publication in a refereed journal, because none analysed the *formal proof* of Weyl - Majorana equivalence under the assumption of *quantitative* validity of the standard model.
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